ME 314 - Engineering Design : Mechanical Components
Lecture 12

Note Title

Chapter 5 - Static Failure Theories

Failure **can mean a component:**

(i) has separated into two or more pieces;

(ii) has become permanently distorted;

(iii) has had its reliability downgraded; or

(iv) has had its function compromised.

Here, we focus on the first two, i.e., we will discuss theories that predict separation and permanent distortion.

Failure under a *static load* is referred to as *static failure*. A static load is a force or moment that is applied in such a manner that its magnitude, point of action, and line of action do not change.

Failure under a *dynamic load* is discussed in Chapter 6.

Ductile and Brittle Failure Mechanisms

The mechanism causing failure of a component also depends on the behavior of the material it is made of. If the material behaves in a ductile manner, significant yielding occurs before fracture or separation. In brittle behavior, fracture occurs without significant strain. Note that if cracks are present in a ductile material, it can suddenly fracture at stress levels well below their yield stress.

5.1 Static Failure of Ductile Materials

If the strain to fracture, for a material is greater than 5%, the material is considered to be ductile. For most ductile metals the strain to fracture is greater than 10%. While ductile materials fracture when they are stressed beyond their ultimate tensile strength, Sut, their failure is considered to occur when they yield at Sy.

Several theories have been proposed to explain failure of ductile materials. Of these only two agree closely with experimental data:

1. **The Distortion Energy (DE) Theory, and**

2. **The Maximum Shear-Stress (MSS) Theory.**

The Distortion Energy Theory (DET)

According to this theory which is also known as the von Mises criterion or the von Mises-Hencky criterion

> **"*Yielding occurs when the distortion energy in a unit volume of material equals the distortion energy in the same volume when uniaxially stressed to yield strength*"**

Recall the expression for strain energy per unit volume (i.e., density):

$$u = \frac{1}{2}\left(\sigma_x \epsilon_x + \sigma_y \epsilon_y + \sigma_z \epsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}\right)$$

In terms of principal stresses and strains:

$$u = \frac{1}{2}\left(\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3\right)$$

Since $\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2 + \sigma_3)$, etc., we have

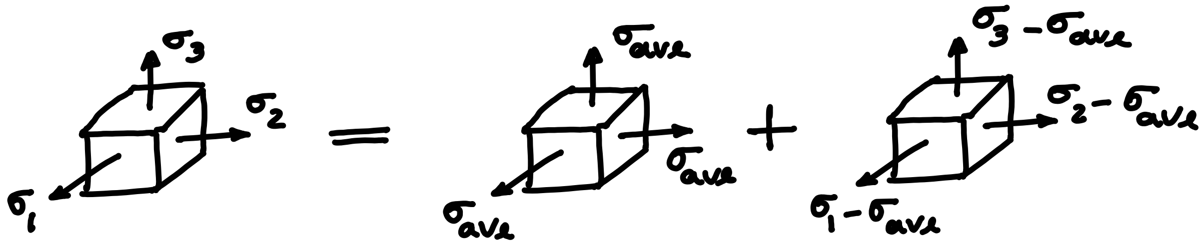
$$u = \frac{1}{2}\sigma_1\left[\frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2 + \sigma_3)\right] + \frac{1}{2}\sigma_2\left[\frac{\sigma_2}{E} - \nu(\sigma_1 + \sigma_3)\right] +$$

$$+ \frac{1}{2}\sigma_3\left[\frac{\sigma_3}{E} - \frac{\nu}{E}(\sigma_1 + \sigma_2)\right]$$

or

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \qquad (1)$$

Now the loading can be divided in two parts:



$\boxed{\text{For isotropic materials}} \Rightarrow$ **This part of stress produces volume change** | **This part produces change in shape or distortion**

where $\sigma_{ave} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ is the average or "hydrostatic" stress. In the text, it is denoted by σ_h. This is in analogy with the pressure in fluids at rest.

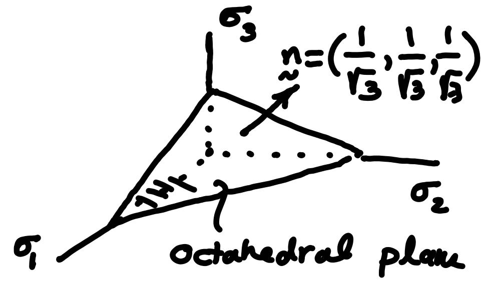
Under a hydrostatic stress, very large amounts of strain energy can be stored in materials without failure. Considering this fact, the cause of failure in ductile materials is the part of loading that causes change in shape (or distortion). To find the distortional part of strain energy, u_d, we note that the volumetric part of strain energy, u_v, can be found from (1) by setting $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_{ave}$ for hydrostatic loading:

Define, the von Mises stress, σ', as

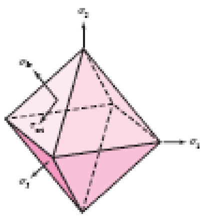
In the case of simple tension test, $\sigma_1 = S_y$, $\sigma_2 = \sigma_3 = 0$, and

Hence, according to the Distortional-Energy Theory (DET):

In terms of octahedral shear stress:



$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$



Various forms of von Mises stress defined by Eq. (4):

$$\sigma' = \sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_3\sigma_1} \quad (5.7\,a)$$

or

$$\sigma' = \sqrt{\frac{1}{2}[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)} \quad (5.7\,b)$$

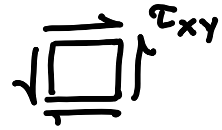
For 2D case with $\sigma_2 = 0$:

With (5-7c), the DE-theory in 2D is given by

For design, we use a factor of safety, N:

where σ' is given either by (5.7a) or (5.7c).

For pure shear **$\sigma_x = \sigma_y = 0$, $\tau_{xy} \neq 0$:**



This result is according to the Distortional-Energy Theory.

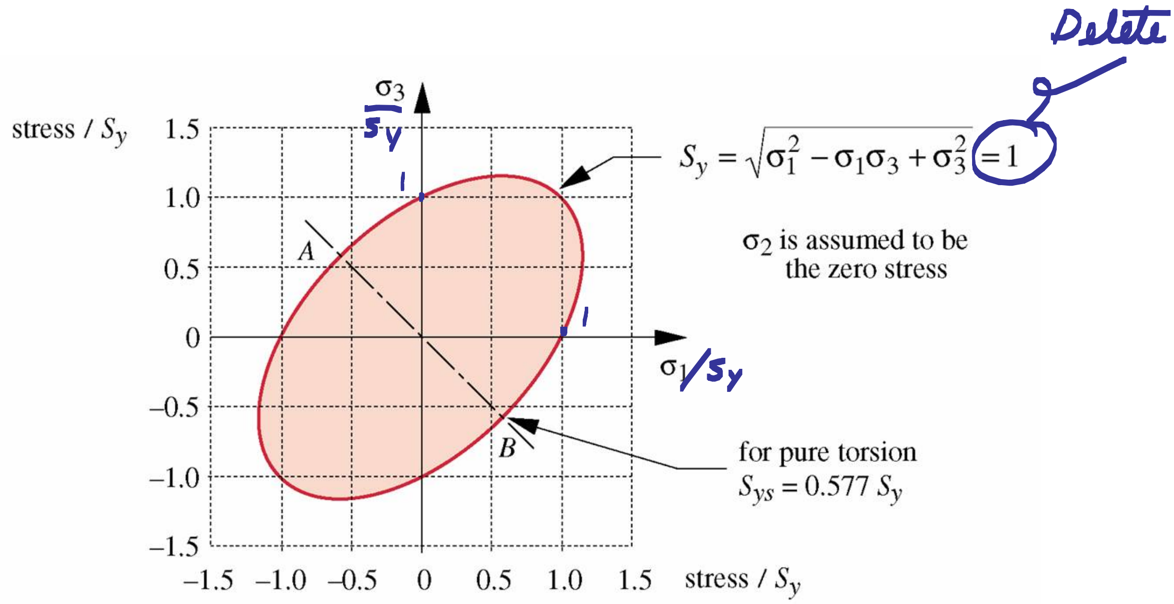


Figure 5-3
The 2-D Distortion-Energy Ellipse Normalized to the Yield Strength of the Material.

The Maximum Shear-Stress (MSS) Theory

According to the MSS theory:

> ***"Failure (yielding) begins whenever the maximum shear stress in any element equals the maximum shear stress in a tension test specimen of the same material when that specimen begins to yield. "***

This is also known as Tresca (1864) Criterion but it was originally proposed by Coulomb (1736-1806) and is the oldest failure theory.